# REPORTS

## **Review of** *Linear Algebra over Commutative Rings*, by Bernard R. McDonald*

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Linear algebra over a field is a highly developed subject which plays a role in most areas of mathematics. It is the topic of many books at both the elementary and advanced levels. Linear algebra and matrix theory over an arbitrary commutative ring certainly plays a role in much current research, but it has not heretofore received a systematic account. Therefore the book under review is most welcome.

The level of sophistication of this book is naturally much higher than that of a typical linear algebra book but it is not a research monograph. It is suitable as a textbook for graduate students or a reference book for research mathematicians, and is quite accessible to nonspecialists.

The author has written the main text in a careful and complete style which students new to the material will appreciate. Secondary topics and topics whose scope precludes a detailed treatment are handled in two ways. Sometimes the major results are stated and the main ideas and techniques of their proofs are discussed, but no proofs are given; other topics are relegated to the exercises, which are an important part of the book. In this way it is possible to give a broad cross-section of current research in a book which is only moderately large. As one would expect, the point of view is entirely algebraic, and the vast area of linear algebra concerned with location of eigenvalues, positive definiteness, etc., is not touched.

There are five chapters. The first gives elementary properties of matrix rings, an overview of various problems concerning the general linear group, and finally studies systems of linear equations.

*Marcel Dekker, New York, 1984.




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The next three chapters are the core of the book and are concerned primarily with projective modules. The second chapter treats free modules, the third treats the endomorphism ring of a projective module, and the fourth treats projective modules. The fundamental theorem of projective geometry is presented in an appendix to the second chapter. These three chapters are well suited to form a preparation for, or an introduction to, algebraic $K$-theory.

The final chapter is devoted to the theory of a single endomorphism. It begins with the tensor, exterior, and symmetric algebras, and then studies the characteristic polynomial of an endomorphism and some related polynomials. Then $\lambda$-rings are defined and several important examples of $\lambda$-rings are introduced. These are used to given Almkvist's description of $K_0(End_R)$, where $End_R$ is the category of pairs $\langle P, \sigma, \rangle$, where $P$ is a finitely generated projective $R$-module and $\sigma$ is an endomorphism of $P$. The final topic covered is the Koszul complex.

There is a large bibliography of more than 600 items.

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